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Technical Report 3

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Abstract

A mathematical model of a simplified atmosphere is described which is suitable for considering certain processes of heat exchange. The model is set up for an almost incompressible atmosphere, that is, the variations in density are neglected except in the term representing gravitational force. Only the dynamics, not the thermodynamics, of the problem is considered. The resulting equations of motion are simplified by perturbation methods regarding the ratio of atmospheric thickness to the radius of the earth as the perturbation parameter. Given a density distribution in the atmosphere as an arbitrary function of latitude and height, the model determines the zonal velocity component and the meridional circulation. A simple numerical example is given to illustrate the type of flow to be expected from the model.

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1. Introduction

Although much effort has been devoted to constructing mathematical and mechanical models of the atmosphere with the aim of explaining the general features of the atmospheric circulation, there is at present no satisfactory manner of linking the observed behavior of the atmosphere to the fundamental sources of energy responsible for that behavior. The ultimate source of energy for atmospheric circulation must be solar radiation. The motion in the atmosphere derives its energy from the unequal distribution of heat produced by the different rates at which heat energy is absorbed from and radiated into outer space at different parts of the atmosphere. The motion in turn furnishes processes of heat exchange which tend to produce a uniform distribution of heat.

The circulation is observed to be near a quasi-equilibrium state at all times so that large surpluses or deficits of heat energy in the atmosphere do not occur. There are, however, significant departures of the circulation from the mean state which do not seem to depend upon a particular distribution of heat energy in the atmosphere but rather suggest a dynamic instability of the basic zonal circulation in at least some of its modes.

The problem of the stability of zonal flow is of great importance and has received a good deal of attention in the past few years. Kuo (1950), for example, has considered the stability problem in considerable detail and has discussed a possible role which stability plays in maintaining and regulating the general circulation. Studies of the stability problem, however, do not give very much insight into how the general circulation is maintained since the conversion of heat into kinetic and potential energy is not considered.

Much of the difficulty in constructing a theory of the general circulation is encountered in choosing a simple, meaningful model in which the thermodynamics of the atmosphere may be included. The mathematical models which have been developed for the study of dynamic stability of zonal circulation are not suitable for the thermodynamic problem because most of them either do not explicitly include the effects of vertical structure in the atmosphere or are applicable only when the thermodynamic processes are negligible. The classical Hadley model is more suitable but is difficult to construct mathematically.

In the classical Hadley model it can be shown by qualitative reasoning that at least three meridional circulation cells must exist to correspond approximately to the observed zonal winds. The subtropic and arctic cells are driven thermally while the middle cell is driven indirectly by the other two cells through lateral friction. The arguments leading to this picture of the circulation consisting of three cells and the dynamics of the middle cell have been given by Rossby. A semi-empirical model of a three cell type has been constructed by Dorodnitsyn, Izvekov, and Schwetz (1939) from the observed temperature distribution in the atmosphere and the surface pressure distribution, using a method developed by Kochin (1935). Rossby (1941) gave an excellent discussion of the Hadley model and pointed out some of the difficulties encountered in attempting to explain certain features of the circulation with this model.

A model of a simplified atmosphere is constructed here in order that some processes of heat exchange can be studied analytically. The purpose of this model is to give the distribution of zonal and meridional velocity components corresponding to a given density field and representing a balance of pressure-gradient, Coriolis, gravitational, and frictional forces. Assuming the density field to be known, the velocity field is determined. Using the energy equation and the equation of state, the distribution of heat sources and sinks necessary to maintain the particular density field in the steady state can be calculated. The thermodynamics of the model can be considered as a separate problem and, therefore, is not taken up in detail here.

2. Simplifying Assumptions

The equations of mass and momentum conservation serve as the basis for the model considered in the following analysis. It is necessary to make simplifying assumptions at the outset in order to obtain a model sufficiently simple for analytical treatment.

The atmosphere is considered to be an almost incompressible fluid covering a smooth earth to a mean depth h which is very small in comparison with the earth's mean radius a . The motion of the atmosphere is considered in the steady state and is assumed to be driven by a known density distribution. It is assumed that the density variations are small enough to be sufficiently accurately represented in their effect on the fluid motion by considering a variable gravitational force. Frictional forces are assumed to arise from Reynolds stresses and are taken into account by introducing a constant, isotropic kinematic eddy viscosity. In adopting an eddy viscosity, it is implicitly assumed that the steady motions are arrived at by an averaging process. The non-linear terms are neglected to simplify the mathematical analysis so that conclusions based on this model would be valid only when these terms are relatively small. The motion is assumed to be independent of longitude since otherwise the motion does not appear to have any obvious physical significance unless surface irregularities of the earth are taken into account.

The coordinates of a point in the atmosphere may be specified by giving the distance z of the point from the geoid measured along the local vertical direction, the latitude φ, and the longitude λ at the geoid. It is assumed that this coordinate system can be sufficiently well approximated by a spherical coordinate system (λ, φ, r) in which gravitational forces act in the radial direction. Here, λ is the longitude measured eastward from a reference meridian, φ the latitude measured from the equatorial plane and r measured from the origin at the center of the earth. The radial distance r of a point at height z above the geoid is approximated by $a + z$, where a is the mean radius of the earth. The velocity components (u, v, w) are chosen so that u is the eastward component, v the northward component, and w the upward component of velocity.

3. Equations of Motion

On the basis of the assumptions above, the equations of motion can be written in the following form

$$2\Omega [\cos\varphi\, w - \sin\varphi\, v] = K[\nabla^2 u - \frac{u}{r^2\cos^2\varphi}] \quad (1)$$

$$2\Omega \sin\varphi\, u = -\frac{1}{\rho r}\frac{\partial p}{\partial\varphi} + K[\nabla^2 v - \frac{v}{r^2\cos^2\varphi} + \frac{2}{r^2}\frac{\partial w}{\partial\varphi}] \quad (2)$$

$$-2\Omega\cos\varphi\, u = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g(\varphi, r) + K[\nabla^2 w - \frac{2w}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial\varphi} + \frac{2\tan\varphi\, v}{r^2}] \quad (3)$$

$$\frac{1}{r\cos\varphi}\frac{\partial\cos\varphi\, v}{\partial\varphi} + \frac{1}{r^2}\frac{\partial r^2 w}{\partial r} = 0 \quad (4)$$

where Ω is the earth's angular velocity, ρ is the density and is considered constant while $g(\varphi, r)$, the unit-mass gravitational force, is modified to include the effects of small variations of the density. The pressure is given by p, and the kinematic eddy viscosity by K. The Laplacian operator in this coordinate system is

$$\nabla^2 = \frac{1}{r^2\cos\varphi}\frac{\partial}{\partial\varphi}[\cos\varphi\frac{\partial}{\partial\varphi}] + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\frac{\partial}{\partial r}]$$

A stream function ψ is introduced for the meridional circulation so that

$$v = \frac{1}{r\cos\varphi}\frac{\partial\psi}{\partial r} \quad (5)$$

$$w = -\frac{1}{r^2\cos\varphi}\frac{\partial\psi}{\partial\varphi} \quad (6)$$

The equations can be expressed more compactly by introducing the quantities M, x, D^2, and G, where

$$M = r\cos\varphi\, u$$

$$x = \sin\varphi$$

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{(1 - x^2)}{r^2} \frac{\partial^2}{\partial x^2}$$

$$G = x \frac{\partial}{\partial r} + \frac{(1 - x^2)}{r} \frac{\partial}{\partial x}$$

and M is the unit-mass relative angular momentum of the zonal flow, and D^2, G are commutative differential operators. The equations become

$$- 2\,\Omega G(\psi) = K\, D^2(M) \tag{7}$$

$$+ 2\,\Omega x \frac{M}{r} = - \frac{(1 - x^2)}{\rho r} \frac{\partial p}{\partial x} + \frac{K}{r} \frac{\partial}{\partial r} [D^2(\psi)] \tag{8}$$

$$- 2\,\Omega \frac{M}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} - g - \frac{K}{r^2} \frac{\partial}{\partial x} [D^2(\psi)] \tag{9}$$

Eliminating the pressure from (8) and (9) yields

$$2\,\Omega G(M) = (1 - x^2) \frac{\partial g}{\partial x} + K\, D^4(\psi) \tag{10}$$

The boundary conditions at the earth's surface are that normal and tangential velocity components are zero. At the upper surface the components of torque are zero. The condition of continuity requires zero net volume transport in the steady state across all latitude circles. In terms of M and ψ , these boundary conditions become

$$\left.\begin{array}{l} \psi = \dfrac{\partial \psi}{\partial r} = 0 \\ M = 0 \end{array}\right\} \text{ at } r = a,$$

and

$$\left.\begin{aligned} \psi = \frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial\psi}{\partial r}\right] &= 0 \\ \frac{\partial}{\partial r}\left[\frac{M}{r^2}\right] &= 0 \end{aligned}\right\} \text{ at } r = a + h.$$

The mathematical problem is specified once g is given as a function of x and r.

4. Dimensional Analysis

Unfortunately, it is difficult to solve equations (7) and (10) regardless of the choice of g. Further simplification of the equations is achieved by dimensional analysis. The equations are converted into non-dimensional form by introducing the non-dimensional quantities α_o, z', M', ψ', g', and p', where

$$\alpha_o = h/a$$
$$z' = z/h \text{ or } r = a\,(1 + \alpha_o z')$$
$$M' = M/aU$$
$$\psi' = \psi/\Psi$$
$$g' = g/\bar{g}$$
$$p' = p/\rho\bar{g}\,h$$

and U, Ψ, $\bar{g}$ are characteristic values of the zonal velocity component, stream function, and gravity respectively.

By choosing

$$\Psi = K\bar{g}\;/4\Omega^2$$
$$U = \alpha_o/\bar{g}\;/2\Omega$$
$$\varepsilon = K/2\Omega\,h^2$$

equations (7) and (10) can be written in the following non-dimensional form, the primes now being omitted

$$- G(\psi) = D^2(M) \tag{11}$$

$$G(M) = (1 - x^2)\frac{\partial g}{\partial x} + \varepsilon^2 D^4(\psi) \tag{12}$$

where the differential operators in non-dimensional form are

$$D^2 = \frac{\partial^2}{\partial z^2} + \alpha_o^2 \frac{(1 - x^2)}{(1 + \alpha_o z)^2}\frac{\partial^2}{\partial x^2}$$

$$G = x\frac{\partial}{\partial z} + \alpha_o \frac{(1 - x^2)}{(1 + \alpha_o z)}\frac{\partial}{\partial x}$$

The order of magnitude of the various terms can be estimated by choosing representative values of the dimensional constants. For example, the values

$$\begin{aligned} a &= 6371 \text{ km} \\ h &= 10 \text{ km} \\ K &= 10^5 \text{ cm}^2\text{sec}^{-1} \\ g &= 10^3 \text{ cm sec}^{-2} \\ \rho &= 10^{-3} \text{ g cm}^{-3} \\ \Omega &= 7.3\times10^{-5} \text{ sec}^{-1} \end{aligned}$$

yield the following characteristic constants

$$\begin{aligned} U &= 110 \text{ m sec}^{-1} \\ \Psi &= 5\times10^9 \text{ m}^3\text{sec}^{-1} \\ \alpha_o &= 0.0016 \\ \varepsilon &= 0.0007 \end{aligned}$$

5. Perturbation Equations

The investigation of equations (11) and (12) can be simplified considerably by regarding α_o as a perturbation parameter and expanding the solutions and the differential equation as perturbation series in α_o.

The functions M, ψ, and g are expanded as power series in α_o

$$M = M_o + \alpha_o M_1 + \alpha_o^2 M_2 + \dots\dots\dots$$

$$\psi = \psi_o + \alpha_o \psi_1 + \alpha_o^2 \psi_2 + \dots\dots\dots$$

$$g = g_o + \alpha_o g_1 + \alpha_o^2 g_2 + \dots\dots\dots$$

These series are substituted into equations (11) and (12) and all terms are collected as coefficients of terms in a power series in α_o. Considering α_o as an arbitrary parameter, the power series can have a sum function identically zero only if all the coefficients of powers of α_o are zero. This condition leads to an infinite set of differential equations for M_j and ψ_j. In the following analysis only the equations corresponding to the zero and first order terms in α_o are considered.

The equations of zero-order in α_o are

$$\varepsilon^2 \frac{\partial^4 \psi_o}{\partial z^4} - x \frac{\partial M_o}{\partial z} = -(1 - x^2) \frac{\partial g_o}{\partial x} \tag{13}$$

$$\frac{\partial^2 M_o}{\partial z^2} + x \frac{\partial \psi_o}{\partial z} = 0 \tag{14}$$

with the boundary conditions

$$\left.\begin{aligned} \psi_o = \partial\psi_o/\partial z &= 0 \\ M_o &= 0 \end{aligned}\right\} \text{ at } z = 0,$$

and

$$\left.\begin{aligned} \psi_o = \partial^2\psi_o/\partial z^2 &= 0 \\ \partial M_o/\partial z &= 0 \end{aligned}\right\} \text{ at } z = 1.$$

The equations of first-order in α_o are

$$\varepsilon^2 \frac{\partial^4\psi_1}{\partial z^4} - x\frac{\partial M_1}{\partial z} = -(1 - x^2)\left[\frac{\partial M_o}{\partial x} + \frac{\partial g_1}{\partial x}\right] \tag{15}$$

$$\frac{\partial^2 M_1}{\partial z^2} + x\frac{\partial\psi_1}{\partial z} = -(1 - x^2)\frac{\partial\psi_o}{\partial x} \tag{16}$$

with the boundary conditions

$$\left.\begin{aligned} \psi_1 = \partial\psi_1/\partial z &= 0 \\ M_1 &= 0 \end{aligned}\right\} \text{ at } z = 0,$$

and

$$\left.\begin{aligned} \psi_1 = 0,\ \partial^2\psi_1/\partial z^2 &= 2\,\partial\psi_o/\partial z \\ \partial M_1/\partial z &= 2M_o \end{aligned}\right\} \text{ at } z = 1.$$

The perturbation equations for the pressure can be obtained in a similar manner from equations (8) and (9) after converting them into non-dimensional form.

$$\frac{\partial p_o}{\partial z} = -g_o \tag{17}$$

$$(1 - x^2)\frac{\partial p_o}{\partial x} = -xM_o + \varepsilon^2\frac{\partial^3\psi_o}{\partial z^3} \tag{18}$$

$$\frac{\partial p_1}{\partial z} = - M_o - g_1 \tag{19}$$

$$(1 - x^2) \frac{\partial p_1}{\partial x} = - xM_1 + \varepsilon^2 \frac{\partial^3 \psi_1}{\partial z^3} \tag{20}$$

6. Solutions of the Zero-order Equations.

Solutions to the zero-order equations can be obtained in closed form for the special case that $\partial g_o/\partial x$ is a function of x only; so for the purposes of studying simple flows predicted by the present model, this special case is considered in detail. The more general case where $\partial g_o/\partial x$ is a function of both z and x can be solved in terms of an integral over a Green's function or by boundary-layer methods.

The solutions to the zero-order equations may be written in the form

$$\psi_o = - \frac{(1 - x^2)}{x^2} \frac{\partial g_o}{\partial x} \phi\,(z,k) \tag{21}$$

$$M_o = \frac{(1 - x^2)}{x} \frac{\partial g_o}{\partial x} \Phi\,(z,k) \tag{22}$$

where

$$\phi(z,k) = 1 - \sum_{j=1}^{4} A_j \exp\,(\lambda_j z) \tag{23}$$

$$\Phi\,(z,k) = \int_o^z \phi(\tau,k)\, d\tau \tag{24}$$

and λ_j $(j = 1,2,3,4)$, represent the four roots of the equation

$$\lambda_j^4 = - x^2/\varepsilon^2 = - 4\, k^4.$$

The roots are

$$\lambda_1 = -k(1-i) \qquad \lambda_3 = +k(1+i)$$

$$\lambda_2 = -k(1+i) \qquad \lambda_4 = +k(1-i)$$

where $$k = \left[\frac{x}{2\varepsilon}\right]^{1/2} \qquad i = (-1)^{1/2}$$

The function $\emptyset(z,k)$ can be found by solving the four simultaneous algebraic equations arising from the boundary conditions for the coefficients A_j. The function is found to be

$$\emptyset(z,k) = 1 - \left\{\frac{2}{\sinh 2k - \sin 2k}\right\} \Big\{ [\cos k \cosh k(1-z) \sin kz - \cosh k \sinh kz \cos k(1-z)]$$
$$+ [\cosh k \cos k - \sinh k \sin k][\cosh k(1-z) \sin k(1-z)]$$
$$- [\cosh k \cos k + \sinh k \sin k][\sinh k(1-z) \cos k(1-z)] \Big\} \qquad (25)$$

There are two cases for which this expression is closely approximated by a much simpler form:

Case I. $k > 5$

$$\emptyset(z,k) \sim 1 - (2)^{\frac{1}{2}} \exp(-kz) \cos(kz - \pi/4) - \exp(-k(1-z)) \cos k(1-z)$$

Case II. $k \leq 1$

$$\emptyset(z,k) \sim \frac{(kz)^2(1-z)(3-2z)}{16}$$

Profiles of $\Phi(z,k)$ and $\emptyset(z,k)$ are given for several values of k in fig. 1 and fig. 2 respectively.

The zero-order pressure distribution can be calculated from equations (17) and (18). If $\partial g_o/\partial x$ is a function of x only, $g_o(z,x)$ may be written as

$$g_o(z,x) = g_{o1}(z) + g_{o2}(x)$$

and from equation (17), the pressure p_o, as

$$p_o(z,x) = p_{o1}(z) + g_{o2}(x)\,[1 + \eta(x) - z] \qquad (26)$$

The equation for $\eta(x)$ is obtained by substituting from (21), (22) and (26), the expressions for ψ_o, M_o, and p_o respectively, into equation (18). The resulting equation is

$$\frac{\partial g_{o2}\eta}{\partial x} = -\frac{\partial g_{o2}}{\partial x}\left[1 - \frac{\cosh 2k + \cos 2k - 2\cosh k \cos k}{k(\sinh 2k - \sin 2k)}\right]. \qquad (27)$$

The boundary condition for $\eta(x)$ is

$$\int_o^1 \eta(x)\,dx = 0.$$

The function $\eta(x)$ represents the departure of the upper surface from the surface $z = 1$ necessary to bring the pressure-gradient forces into equilibrium with Coriolis and frictional forces.

The horizontal pressure gradient can be expressed as follows

$$\frac{\partial p_o}{\partial x} = \frac{\partial}{\partial x}\,[\,g_{o2}(1 + \eta - z)]$$

$$= \frac{\partial g_{o2}}{\partial x}\left[\frac{\cosh 2k + \cos 2k - 2\cosh k \cos k}{k\,(\sinh 2k - \sin 2k)} - z\right] \qquad (28)$$

It is clear that the horizontal component of the pressure gradient must change sign in the lower frictional layer and will be zero along the surface

$$z = \frac{\cosh 2k + \cos 2k - 2 \cosh k \cos k}{k (\sinh 2k - \sin 2k)} \tag{29}$$

for any choice of $\partial g_o/\partial x$ which depends only on x. Along this surface the Coriolis and frictional forces are of equal magnitude.

7. Surface Torque

It is of interest to note that equation (14) can be integrated with respect to z directly. The resulting equation is

$$\frac{\partial M_o}{\partial z} + \varkappa\psi_o = 0 \tag{30}$$

This implies that the meridional transport at a given latitude is directly proportional to the vertical gradient of the zonal angular momentum. Since $\partial M_o/\partial z$ is also proportional to the torque exerted by the zonal flow, equation (30) may be interpreted to mean that the volume transport per unit time in the meridional direction below a certain height is proportional to the torque exerted by the zonal flow above that height. Since there is no volume transport below the surface z = 0, the zonal flow given by the zero-order equations cannot exert any torque on the earth's surface in the steady state.

The solutions to the first-order equations can be obtained in terms of a Green's function, but as these solutions cannot be expressed in the simple form found from the zero-order solutions, they are not considered in detail here. However, it is possible to draw some conclusions about the torque on the earth's surface from the first-order differential equations. Integrating equation (16) with respect to z throughout the atmosphere gives the result that the surface torque must be

$$T_s = -\frac{\partial}{\partial x}\left[\frac{(1-x^2)}{x} M_o\right]_{z=1} \qquad (31)$$

Since $M_o(1,x)$ is zero at the poles and at the equator, the torque must change sign in the interval $0 < |x| < 1$. This, in turn implies that the zonal flow must reverse itself in some region between the poles and the equator. As this reversal of the zonal flow is contributed by the first-order equations, and consequently is small relative to the zero-order terms, the region of reversal will not extend beyond the lower layer of frictional influence.

8. A Simple Numerical Example

Most of the features of this model can be illustrated by considering a numerical example in which $\partial g_o/\partial x$ is assumed to be a simple polynomial in x. Since g_o is functionally dependent on density in the approximation adopted in this model, assuming $\partial g_o/\partial x$ to be known is equivalent to assuming the distribution of the horizontal density gradient throughout the atmosphere to be known. The density gradient is chosen to be anti-symmetrical about the equator, to vanish at poles, and to be proportional to

$$\frac{\partial g_o}{\partial x} = \beta x^3(1 - x^2) \qquad (32)$$

The effects of friction are measured by the dimensionless constant ε, which in this example is chosen to have the value $\varepsilon = 10^{-3}$ to simplify numerical calculations.

The corresponding zero-order solutions are

$$\psi_o = -\beta x(1 - x^2)^2 \phi(z,k) \tag{33}$$

$$M_o = \beta x^2 (1 - x^2)^2 \Phi(z,k) \tag{34}$$

$$g_o = f(z) + \frac{\beta}{12} x^4 (3 - 2x^2) \tag{35}$$

where β is a dimensionless constant, and $f(z)$ is an arbitrary function of z.

Meridional cross-sections have been drawn showing isotachs of the zonal velocity component in figure 3, and streamlines of the meridional circulation in figure 4. The normalization factor for the zonal velocity component is $u_{max} = 19.0\ \beta$ m sec^{-1}, and for the stream function, $\Psi = 6.85 \times 10^9 \beta$ m^{3}sec^{-1}. The maximum values of the velocity components obtained in the numerical example are:

$u_{max} = 19.0\ \beta$ m sec^{-1}

$v_{max} = 0.59\ \beta$ m sec^{-1} in the upper frictional layer

$v_{max} = 0.38\ \beta$ m sec^{-1} in the lower frictional layer

$w_{max} = 0.0014\ \beta$ m sec^{-1}.

9. Discussion

The simple example of zonal and meridional flow given above illustrates the type of flow to be expected from this model. The flow pattern is essentially different from the flow to be expected on the basis of the classical Hadley model since the transport of relative angular momentum by the meridional circulation is not taken into account. The easterly winds near the equator predicted by the model are not due to horizontal meridional currents as suggested by the Hadley model but are due to a net vertical motion throughout the atmosphere in the equatorial regions. The easterly winds have an appreciable horizontal extent only in the case that meridional density gradients are very small in the equatorial regions. Thus, a density gradient varying as x or x^2 in the vicinity of the equator would require very strong east winds within 2 or 3 degrees of the equator. Since the derivation of the perturbation equations implicitly assumes that no such rapid changes in velocity occur, the density gradient in the example treated above is chosen to vary as x^3 near the equator.

The example given above is not intended to resemble the actual circulation in the atmosphere. It would not be difficult to choose a density distribution which would give a more realistic zonal velocity profile; however, such a choice could not be justified from the dynamics of the model alone and would require the investigation of the thermodynamic processes associated with the motion for its justification.

10. Plans for Future Work

The basic purpose of pursuing this analysis was to construct a model in which heat exchanges could be considered analytically.

The model described here is considered as the first approximation to a more realistic model and would have to be generalized according to the problem under consideration. Many generalizations can be suggested; for example, it may be necessary to take into account the compressibility of the atmosphere before significant thermodynamic processes can be considered. Also the effects of lateral stresses can be taken into account by considering a nonisotropic eddy viscosity. Both of these generalizations make the mathematics of the model more complicated but, it is hoped, not impractical. The effects of the non-linear terms would have to be considered seriously as they are not always relatively small and may invalidate conclusions made from the linear models. It is not proposed to study models with longitudinal dependence because in such models time dependent terms would have to be considered and the analysis would be much more complicated.

The immediate plan is to apply this model to the investigation of the dynamics and thermodynamics of the Antarctic Circumpolar Current because only minor changes of the present model are necessary in applying it to the ocean. In particular it would be necessary to compute the solutions to the zero-order equations corresponding to a given distribution of surface torque exerted by the prevailing winds.

Acknowledgment

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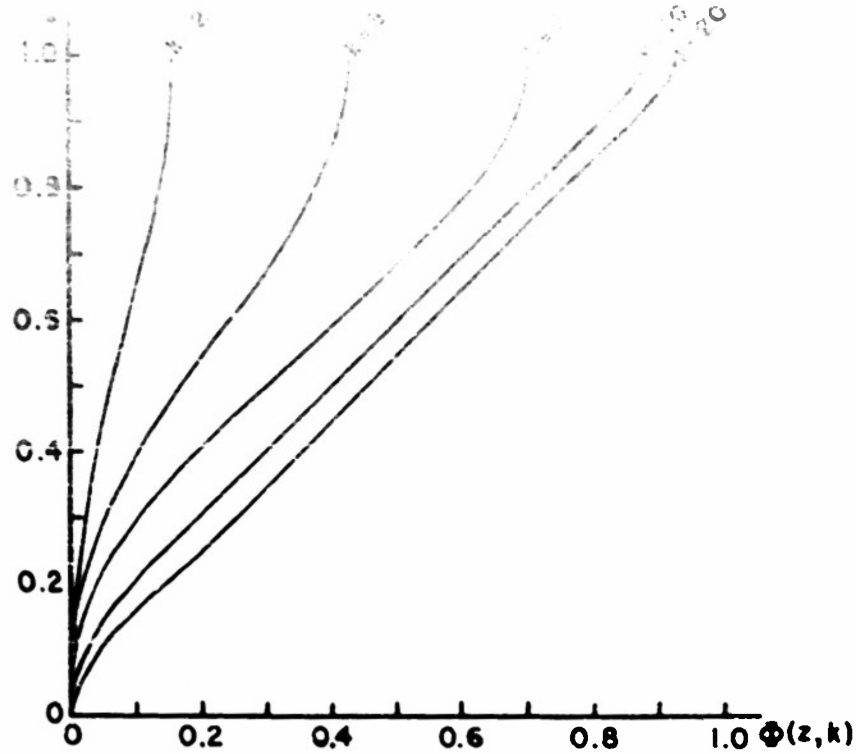

Fig. 1. The function $\phi(z,k)$, defined by eq. (25), is plotted for several values of k. For the numerical example discussed, this function is proportional to the zonal velocity component along a vertical line through the atmosphere.

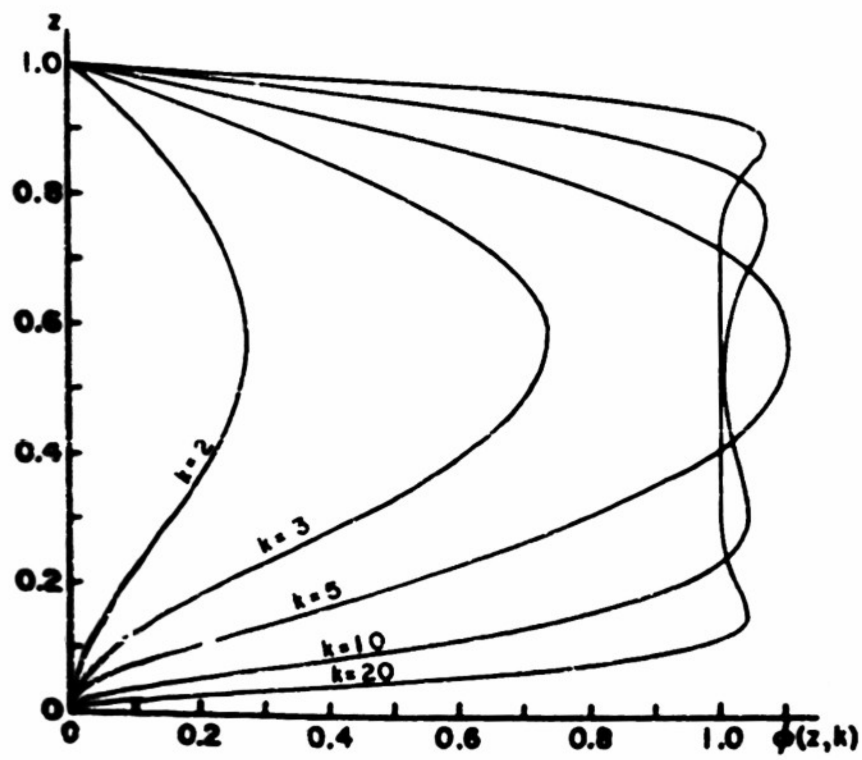

Fig. 2. The function $\phi(z,k)$, defined by eq. (26), is plotted for several values of k. For the numerical example discussed, this function is proportional to the stream function of the meridional circulation along a vertical line through the atmosphere.

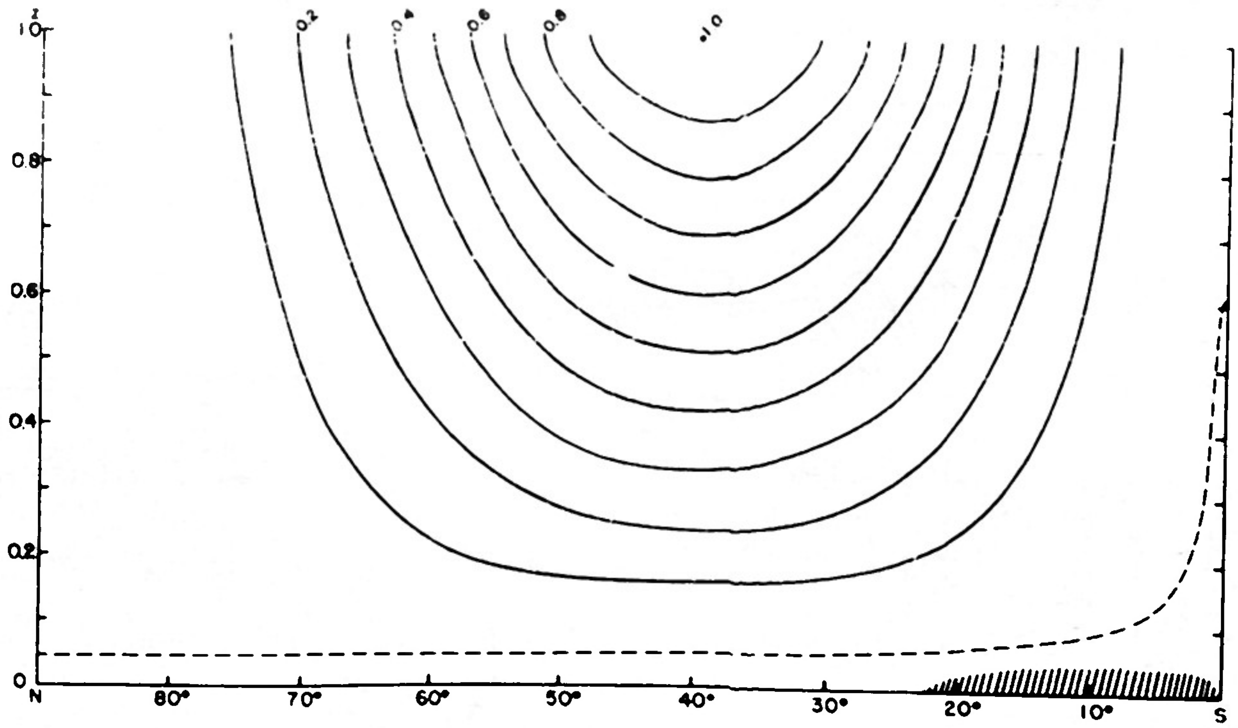

Fig. 3. A meridional cross-section showing isotachs of the zonal velocity component (solid lines) normalized to unit maximum speed at z = 1. The surface along which the horizontal pressure gradient vanishes is marked by a broken line. The hatched region indicates the horizontal extent of the easterly winds as given by the first-order equations.

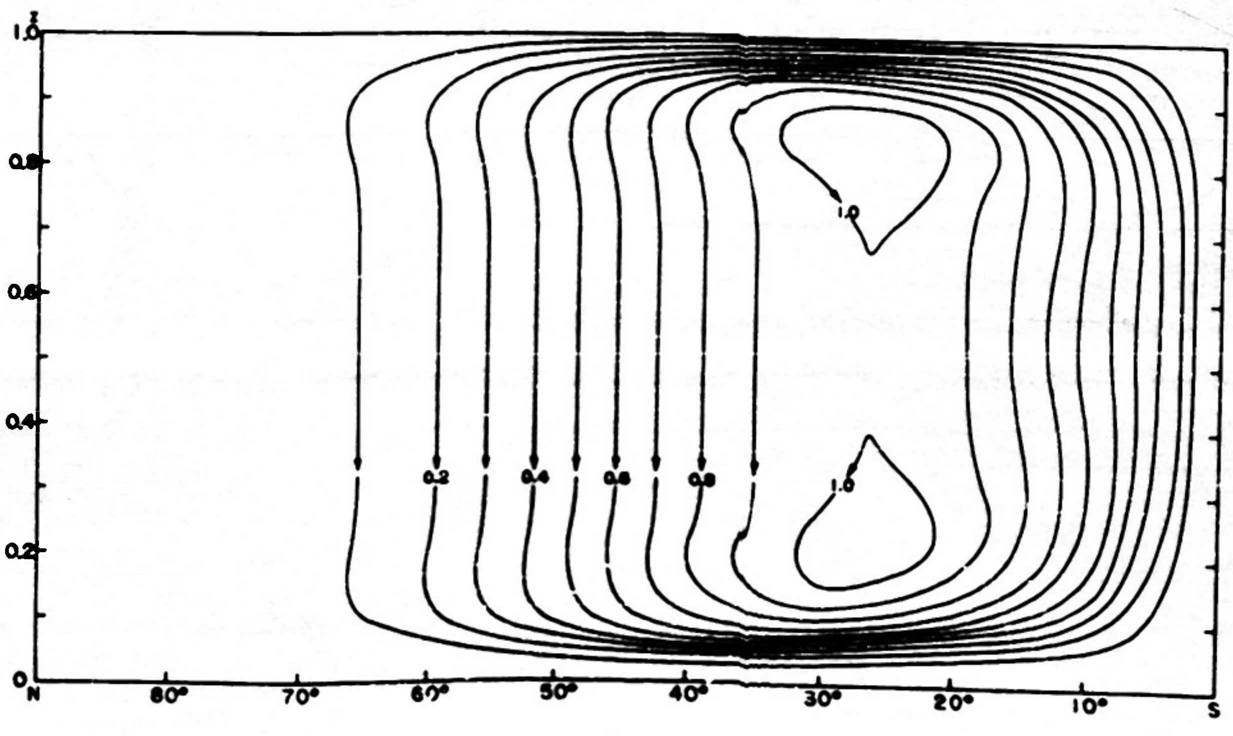

Fig. 4. A meridional cross-section showing streamlines of the meridional circulation normalized to unit maximum value of the stream function at z = 0.5.

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